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Technical Notes

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Sensitivity Analysis of Three-Gate Lifetime Pressure- and Temperature-Sensitive Paint Measurements

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I. Introduction

PRESSURE-SENSITIVE paint (PSP) has established itself as an important technique for measuring pressure distributions on test articles in aerodynamic test facilities. The principle of the technique has been reviewed extensively elsewhere [1,2]. In its most common form, a paint containing luminescent probe molecules is applied to a test article, which is then illuminated by short-wavelength light to produce a pressure-dependent emission signal at a longer wavelength. Measurements can be performed using continuous or pulsed illumination. In the latter case, images of the test article are captured using digital cameras that are gated with respect to the train of illumination pulses. By suitable selection of the pulse repetition frequency, pulse duration, and start and end times of the camera gates, the ratio of the signals collected during two gates becomes a function of pressure, thereby forming the basis for a pressure measurement.

Among the complications of such PSP measurements is the fact that signal ratios usually depend on temperature to some degree [3]. Variations in surface temperature thus tend to produce pressure measurement errors. Recent work on lifetime-based PSP suggests that, given a suitable paint, measurement at a third gate might be added to compensate for this unwanted temperature dependence and to perform a temperature measurement in its own right [4–10]. In Sec. II, this previous work is cast in a form that lends itself to a new analysis approach. The new approach is developed in Sec. III to guide future work in this area and has, in fact, already been used for this purpose [9,10]. In Sec. IV, the model from Sec. III is applied to published data on current paints.

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II. Review of Gated Measurement Concept

Figure 1 shows an example of a fluorescence response, $S(t;P,T)$, that might be the result of excitation of a paint by a nominally square pulse with shape $p(t)$. The exact shape of the response (especially that of the decay portion) depends on the pressure P and temperature T of the surface, as well as the excitation pulse shape [5,6,11]. High pressures produce faster decays than lower pressures. To a lesser extent, high temperatures also produce faster decays than lower temperatures. Because PSP is primarily an image-based technique, it is not practical to measure the detailed shape of the response function, except in calibration measurements. Instead, lifetime-based PSP measurements rely on integrating the signal under gated portions of the response function. The single-pulse signal, s_i, for a gate with index i may thus be written as

$$s_i = \int_{t_i^{(1)}}^{t_i^{(2)}} S(t;P,T)\,\mathrm{d}t \tag{1}$$

Here $t_i^{(1)}$ and $t_i^{(2)}$ denote the start and end times of gate i (see Fig. 1). Let N_i represent the number of pulses over which the gate-i signal is integrated, and let N represent the total number of pulses (i.e., $N = N_1 + N_2$ for a two-gate measurement and $N = N_1 + N_2 + N_3$ for a three-gate measurement). The pulse-integrated signal for gate i may then be written as

$$S_i = N f_i s_i \tag{2}$$

Here, $f_i \equiv N_i/N_i$ represents the fraction of the total measurement time that is devoted to collecting signal at the ith gate. (As shown by Bell, it should not be considered a foregone conclusion that the same number of pulses should be devoted to each gate [12].)

Based on Eq. (2), the ratio between two signals, S_i and S_j, acquired at separate gates i and j, may be written as

$$R_{ij} \equiv \frac{S_i}{S_j} = \frac{f_i s_i}{f_j s_j} \equiv \frac{f_i}{f_j} r_{ij} \tag{3}$$

Here r_{ij} may be thought of as the intrinsic single-pulse ratio for the two gates.

If the paint response can be characterized by a single time constant τ, the ratio R_{ij} may be written explicitly in terms of this time constant. It is then possible to define the relative sensitivity $(\mathrm{d}\tau/\tau)/(\mathrm{d}R_{ij}/R_{ij})$. Several authors have followed this approach to optimize the operational parameters of a two-gate PSP measurement [12–14]. Here a more general approach is followed that allows for the presence of multiple time constants in the fluorescence response [3,11,15,16].

In the generalized approach, the intrinsic pulse ratio r_{ij} from Eq. (3) at a pressure $P_0 + \Delta P$ and a temperature $T_0 + \Delta T$ near an operating point (P_0, T_0) is written as

$$r_{ij} = r_{ij}^{(0)}[1 + p_{ij}\Delta P + t_{ij}\Delta T] \tag{4}$$

Here $r_{ij}^{(0)}$ is the ratio at the operating point and p_{ij} and t_{ij} are the relative partial derivatives of this ratio with respect to pressure and temperature, also at the operating point.

Equation (4) is not meant to describe accurately the calibration equation for a PSP measurement over the entire range of pressures and temperatures of interest. This generally requires the inclusion of quadratic and cubic terms in addition to the linear terms from Eq. (4).

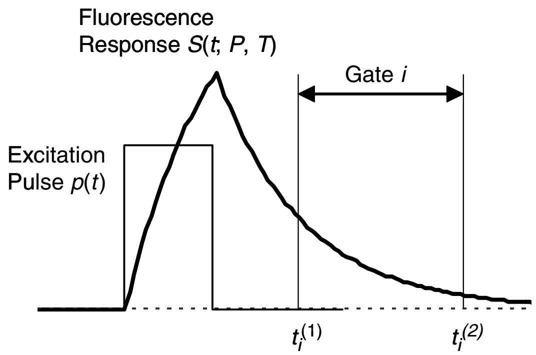

Fig. 1 Schematic of gated measurement.

Rather, the linear form of Eq. (4) is used in Secs. III and IV to approach the optimization of a two-gate or three-gate PSP measurement near an operating point (P_0, T_0).

From Eq. (4) it follows that the temperature sensitivity of a PSP measurement can be defined as

$$\frac{\Delta P}{\Delta T} = -\frac{t_{ij}}{p_{ij}} \tag{5}$$

This sensitivity is typically quoted as percent pressure per unit temperature at some operating pressure P_0 and is denoted here as

$$\eta_{ij} \equiv \frac{1}{P_0}\frac{\Delta P}{\Delta T} \cdot 100\% = -\frac{1}{P_0}\frac{t_{ij}}{p_{ij}} \cdot 100\% \tag{6}$$

Values of this relative temperature sensitivity η_{ij} are typically on the order of 1% per degree Celsius at 1 atm for a non-temperature-compensated paint.

III. Three-Gate vs Two-Gate Measurements

We now take a conceptual look at how a temperature error produces a pressure error in a two-gate PSP measurement and how this error might be mitigated by a three-gate measurement.

Let ΔS_i and ΔS_j denote errors in the signals S_i and S_j attributable to noise or systematic effects and let ΔP and ΔT denote the corresponding errors in pressure and temperature. From Eqs. (3) and (4) it follows that these errors are related, to first order, by

$$\frac{\Delta S_i}{S_i} - \frac{\Delta S_j}{S_j} = p_{ij}\Delta P + t_{ij}\Delta T \tag{7}$$

From this it follows [cf. Eq. (5)] that a two-gate PSP measurement has a bias error given by

$$\Delta P^{(2)} = -\frac{t_{12}}{p_{12}}\Delta T \tag{8}$$

Here ΔT is the (generally unknown) amount by which the assumed temperature differs from the actual temperature. The corresponding noise-induced error is calculated by assuming (cf. Bell [12] and Goss [13]) that the signals S_i and S_j are shot noise limited (this is the best possible scenario [17]), with a relative variance given by

$$\langle(\Delta S_i/S_i)^2\rangle = \frac{1}{GS_i} = \frac{1}{GNf_i s_i} \qquad (j \text{ likewise}) \tag{9}$$

Here G is the gain of the camera in units of photoelectrons per count. As shown in [18], the root-mean square (rms) pressure error in a two-gate measurement may then be written as

$$\sigma_P^{(2)} = \frac{1}{|p_{12}|\sqrt{GN}}\left(\frac{1}{\sqrt{s_1}} + \frac{1}{\sqrt{s_2}}\right) \tag{10}$$

Here N is the total number of pulses among the two gates, p_{12} is the relative partial pressure sensitivity at the operating point, and s_1 and s_2 are the single-pulse signals from Eq. (1). The result from Eq. (10) assumes that the pulse allocation fractions f_1 and f_2 have been selected optimally. Figure 2 shows graphically how the temperature-induced bias error $\Delta P^{(2)}$ and the noise error $\sigma_P^{(2)}$ arise in a two-gate measurement.

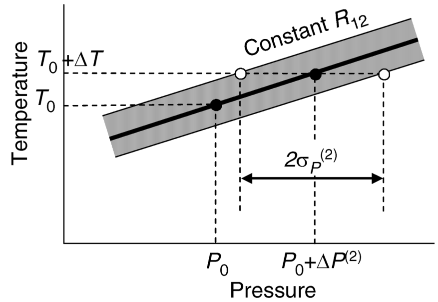

Fig. 2 Bias error and noise-induced errors in two-gate PSP.

In a three-gate PSP measurement, the bias error attributable to unknown temperature is removed (because the temperature is measured implicitly) and the rms pressure error becomes (see again [18])

$$\sigma_P^{(3)} = \frac{1}{|p_{12}t_{13} - p_{13}t_{12}|\sqrt{GN}}\left(\frac{|t_{13} - t_{12}|}{\sqrt{s_1}} + \frac{|t_{13}|}{\sqrt{s_2}} + \frac{|t_{12}|}{\sqrt{s_3}}\right) \tag{11}$$

Here p_{12} and t_{12} are the relative partial pressure and temperature sensitivities for the signal ratio r_{12}, and p_{13} and t_{13} are the relative partial pressure and temperature sensitivities for the signal ratio r_{13}. As is the case for the two-gate result from Eq. (10), the result from Eq. (11) assumes that the pulse allocation fractions (in this case, f_1, f_2, and f_3) have been selected optimally. Figure 3 illustrates graphically how the noise error $\sigma_P^{(3)}$ arises in a three-gate measurement.

Whether the three-gate rms error from Eq. (11) is smaller than the two-gate bias error from Eq. (8) depends on several factors—specifically, the temperature sensitivity of the two-gate measurement (which can be calibrated), the temperature error in the two-gate scheme (which is generally unknown), and the number of pulses over which the three-gate measurement is performed.

Comparison of the three-gate rms error from Eq. (11) to the two-gate rms error from Eq. (10) is possible if three assumptions are met: 1) The same number of pulses, N, is used in both cases; 2) the single-pulse signals s_1, s_2, and s_3 in Eqs. (10) and (11) are roughly balanced in the sense that (for some common s_0) $s_1 = s_2 = s_0/2$ in Eq. (10) and $s_1 = s_2 = s_3 = s_0/3$ in Eq. (11); and 3) $t_{13} > t_{12} > 0$. In terms of the temperature sensitivities η_{12} and η_{13} for the two ratios from Eq. (6), it then follows that

$$\frac{\sigma_P^{(3)}}{\sigma_P^{(2)}} \cong \left|\frac{\eta_{13}}{\eta_{12} - \eta_{13}}\right| \equiv \Sigma_{32} \tag{12}$$

This result suggests that, barring large temperature errors in the two-gate scheme, a necessary condition for a three-gate measurement to outperform a two-gate measurement is that the relative temperature sensitivities η_{12} and η_{13} in the three-gate scheme must be substantially different from each other, so that a small value of Σ_{32} is obtained (say, on the order of unity or less). This corresponds to a small overlap region of the contour bands for R_{12} and R_{13} in Fig. 3.

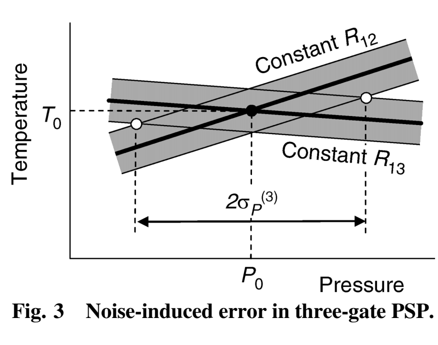

Fig. 3 Noise-induced error in three-gate PSP.

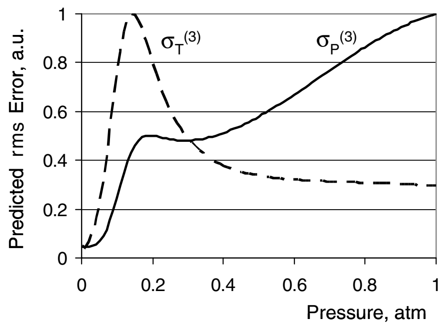

Fig. 4 Predicted three-gate rms pressure and temperature errors at room temperature for [6].

IV. Application of Theory to Current Paints

In Table 1 of [18], values of the two-gate sensitivity factor η_{ij} from Eq. (6) and the three-gate-to-two-gate rms ratio factor Σ_{32} from Eq. (12) are listed for a number of paint systems that have so far been discussed in the literature. Of particular interest is the work by Mitsuo et al., [5,6] in which a three-gate measurement scheme has been demonstrated for a paint with a PtTFPP probe embedded in poly-IBM-co-TFEM polymer. Even so, a factor Σ_{32} on the order of 3 is obtained for this paint in [18], suggesting that random error due to photon shot noise is about 3 times higher in the three-gate scheme than in an equivalent two-gate scheme. Also, it is shown in [18] that PtTFPP in fluoro-isopropyl-butyl (FIB) does *not* lend itself to a three-gate scheme, contrary to the claim by Watkins [7]. This result should not be surprising, in light of the fact that this paint has been described as "ideal" by its developers, in the sense that its temperature sensitivity is independent of pressure [19]. Use of PtTFPP in FIB for a three-gate measurement has also been dismissed by Goss et al. [9,10].

Several authors have also discussed paints in which two probe species are used to introduce a second time constant into the paint deliberately. In the case of Hradil et al. [8], this was a long-lived thermographic phosphor (MFG). The disadvantage of this approach is that the pulse repetition frequency must be decreased, resulting in a longer measurement time. Goss et al. have taken the opposite approach by adding a short-lived laser dye (such as Pyridine 2) to the conventional PtTFPP in FIB system [9,10]. This approach appears to hold good promise for a three-gate measurement system, though the efforts of Goss et al. appear to be focused on developing a temperature-compensated two-gate measurement.

A disadvantage of adding a second probe molecule into the paint is that additional measurement may be required (the so-called ratio-of-ratios approach) to calibrate for uneven spatial distributions of the two probe molecules [12]. Further research is required to decide if the added complexity of such a measurement scheme is offset by the advantages of temperature compensation.

Finally, it should be remembered that it is generally not possible to express the merits of a certain paint system and gating scheme by a single number across the entire measurement range. For example, in [6], Mitsuo et al. report that reasonable pressure and temperature measurements were obtained at room temperature over a range of pressures (15–80 kPa), but that poor results were obtained near 20 kPa. This result was replicated by the author using the model from Sec. III, using as input the calibration parameters for the paint, graciously supplied by Mitsuo. Figure 4 shows results from this calculation. The agreement between Fig. 4 and the data from Table 1 in [6] is not exact, but Fig. 4 shows that, indeed, for the paint system and gating parameters selected, increased error levels are predicted near 20 kPa (i.e., 0.2 atm).

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